Magnetic-flux-controlled giant Fano factor for the coherent tunneling through a parallel double-quantum-dot

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Abstract

We report our studies of zero-frequency shot noise in tunneling through a parallel-coupled quantum dot interferometer by employing number-resolved quantum rate equations. We show that the combination of quantum interference effect between two pathways and strong Coulomb repulsion could result in a giant Fano factor, which is controllable by tuning the enclosed magnetic flux.

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Quantum shot noise in nanosystem far from equilibrium has become currently active issue since it characterizes the degree of correlation between charge transport events, which can not be obtained by measuring mean current alone.[1] In particular, it is specially interesting to search for mesoscopic devices having super-Poissonian noise feature.[2,3,4,5,6,7,8] For instance, a single quantum dot (QD) with multilevel connected to two leads was recently reported to show enhanced noise by analyzing its full counting statistic, in which quantum interference between multi-states was not taken into account.[9]

In this letter, we study the shot noise of first-order resonant tunneling through a two-level system, a two parallel-coupled QDs, under an extremely large bias-voltage with emphasis on the quantum interference effect. The theoretical model is depicted in Fig. 1, in which both of two QDs 1 and 2 are connected to the left and right leads via the tunneling matrix element $V_{\eta j}$ ($\eta = \{L, R\}$ and $j = \{1, 2\}$) describing the coupling between QD j and lead η. For simplicity, we assume $V_{L1} = V_{R2}$ and $V_{L2} = V_{R1}$, and the strength of tunneling $\Gamma = 2\pi\varrho_0|V_{L1(R2)}|^2$ (ϱ_0 is the flat density of states of the leads) and $\Gamma' = 2\pi\varrho_0|V_{L2(R1)}|^2$ being constant in the wide band limit. Here we also assume that only one spinless energy level ε_j in each dot is involved in transport. Ω denotes dot-dot hopping and a magnetic flux $\varphi \equiv 2\pi\Phi/\Phi_0$ ($\Phi_0 \equiv hc/e$ is the magnetic flux quantum) is applied to penetrate the area enclosed by the two tunneling pathways. Note that this QD Aharonov-Bohm interferometer has been realized in recent experiments.[10,11]

Under the limit of sufficiently large bias-voltage ($V \gg \Gamma, \Gamma', \Omega$), electronic tunneling through this system in first-order picture can be described by the quantum rate equations (QREs) for the dynamic evolution of the reduced density matrix elements of the coupled QDs, $\rho_{ab}(t)$ ($a, b = \{0, 1, 2, d\}$).[12,13,14] The diagonal elements of the reduced density matrix, ρ_{aa}, give the occupation probabilities of the states of the dots, namely: $\rho_{00}(\rho_{dd})$ is the probability of finding both dots unoccupied(occupied), ρ_{11} and ρ_{22} are the probabilities of finding dot 1 and dot 2 occupied, respectively; while the off-diagonal element $\rho_{12} = \rho_{21}^*$ describe the coherent superposition state between two QDs. For the purpose of evaluating the noise spectrum, we introduce the number-resolved density matrices $\rho_{ab}^{(n)}(t)$, meaning that the system is in the electronic state $|a\rangle$ ($a = b$) or in quantum superposition state ($a \neq b$) at time t and meanwhile total n electrons are counted to transfer into the right lead by time t.[12] Obviously, $\rho_{ab}(t) = \sum_n \rho_{ab}^{(n)}(t)$ and the resulting number-resolved QREs at zero temperature

and large bias-voltage in the case of infinite inter-dot Coulomb interaction are:

$$\begin{aligned}\dot{\rho}_{00}^{(n)} &= \Gamma'\rho_{11}^{(n-1)} + \Gamma\rho_{22}^{(n-1)} - (\Gamma+\Gamma')\rho_{00}^{(n)} \\ &\quad + \kappa\sqrt{\Gamma\Gamma'}(e^{-i\varphi/2}\rho_{12}^{(n-1)} + e^{i\varphi/2}\rho_{21}^{(n-1)}), \qquad (1a)\\ \dot{\rho}_{11}^{(n)} &= \Gamma\rho_{00}^{(n)} - \Gamma'\rho_{11}^{(n)} + i\Omega(\rho_{12}^{(n)} - \rho_{21}^{(n)}) \\ &\quad - \frac{\kappa}{2}\sqrt{\Gamma\Gamma'}(e^{-i\varphi/2}\rho_{12}^{(n)} + e^{i\varphi/2}\rho_{21}^{(n)}), \qquad (1b)\\ \dot{\rho}_{22}^{(n)} &= \Gamma'\rho_{00}^{(n)} - \Gamma\rho_{22}^{(n)} - i\Omega(\rho_{12}^{(n)} - \rho_{21}^{(n)}) \\ &\quad - \frac{\kappa}{2}\sqrt{\Gamma\Gamma'}(e^{-i\varphi/2}\rho_{12}^{(n)} + e^{i\varphi/2}\rho_{21}^{(n)}), \qquad (1c)\\ \dot{\rho}_{12}^{(n)} &= i\Omega(\rho_{11}^{(n)} - \rho_{22}^{(n)}) - \frac{1}{2}(\Gamma+\Gamma')\rho_{12}^{(n)} \\ &\quad + \sqrt{\Gamma\Gamma'}e^{-i\varphi/2}\rho_{00}^{(n)} - \frac{\kappa}{2}\sqrt{\Gamma\Gamma'}e^{i\varphi/2}(\rho_{11}^{(n)} + \rho_{22}^{(n)}), \qquad (1d)\end{aligned}$$

together with the normalization relation $\rho_{00}+\rho_{11}+\rho_{22}=1$. The adjoint equation of Eq. (1d) gives the equation of motion for $\rho_{21}^{(n)}$. The parameter $\kappa\sqrt{\Gamma\Gamma'}$ describes the interference in tunneling events through different pathways, in which κ is artificially introduced to describe decoherence [$\kappa = 0(1)$ means full noninterference (interference) between two pathways].[15,16] From these number-resolved QREs, we can readily deduce the usual QREs for the reduced density matrix elements, $\boldsymbol{\rho}(t) = (\rho_{00}, \rho_{11}, \rho_{22}, \rho_{12}, \rho_{21})^T$, as: $\dot{\boldsymbol{\rho}}(t) = \mathcal{M}\boldsymbol{\rho}(t)$, where $\mathcal{M}$ can be easily read from Eqs. (1a)-(1d).

The current flowing through the system can be evaluated by the time change rate of electron number in the right lead (we use $\hbar = e = 1$)

$$I = \dot{N}_R(t) = \frac{d}{dt}\sum_n nP_n(t)\Big|_{t\to\infty}, \qquad (2)$$

where $P_n(t) = \rho_{00}^{(n)}(t) + \rho_{11}^{(n)}(t) + \rho_{22}^{(n)}(t)$ is the total probability of transferring n electrons into the right lead by time t. According to the MacDonald's formula for shot noise,[17] the zero-frequency shot noise with respect to the right lead is defined by $P_n(t)$ as well:[18,19]

$$S(0) = \frac{d}{dt}\left[\sum_n n^2P_n(t) - (tI)^2\right]\Big|_{t\to\infty}. \qquad (3)$$

With the help of Eqs. (1), the current and shot noise can be written as

$$\begin{aligned} I &= \Big[\Gamma'\rho_{11} + \Gamma\rho_{22} + \kappa\sqrt{\Gamma\Gamma'} \\ &\quad \times(e^{-i\varphi/2}\rho_{12} + e^{i\varphi/2}\rho_{21})\Big]\Big|_{t\to\infty}, \qquad (4)\\ S(0) &= \Big\{\Gamma'(2G_{11}+\rho_{11}) + \Gamma(2G_{22}+\rho_{22}) + \kappa\sqrt{\Gamma\Gamma'}\end{aligned}$$

$$\times \left[e^{-i\varphi/2}(2G_{12}+\rho_{12})+e^{i\varphi/2}(2G_{21}+\rho_{21})\right]\Big\}\Big|_{t\to\infty}, \tag{5}$$

with the generating function $G_{ab}(t)$ defined as

$$G_{ab}(t)=\sum_n n\rho_{ab}^{(n)}(t). \tag{6}$$

Employing Eqs. (1), the equations of motion for $\boldsymbol{G}(t) = (G_{00}, G_{11}, G_{22}, G_{12}, G_{21})^T$ are explicitly obtained in matrix form: $\dot{\boldsymbol{G}}(t) = \mathcal{M}\boldsymbol{G}(t) + \mathcal{G}\boldsymbol{\rho}(t)$ with $\mathcal{G}_{12} = \Gamma'$, $\mathcal{G}_{13} = \Gamma$, $\mathcal{G}_{14(5)} = \kappa\sqrt{\Gamma\Gamma'}e^{\mp i\varphi/2}$, and all other elements being zero. Applying Laplace transform to these equations yields

$$\boldsymbol{G}(s)=(s\boldsymbol{I}-\mathcal{M})^{-1}\mathcal{G}\boldsymbol{\rho}(s), \tag{7}$$

where $\boldsymbol{\rho}(s)$ is readily obtained by performing Laplace transform on its equations of motion with the initial condition $\boldsymbol{\rho}(0) = \boldsymbol{\rho}_{st}$ ($\boldsymbol{\rho}_{st}$ denotes the stationary solution of the QREs). Due to inherent long-time stability of the physics system under investigation, all real parts of nonzero poles of $\boldsymbol{\rho}(s)$ and $\boldsymbol{G}(s)$ are negative definite. Consequently, the large-t behavior of the auxiliary functions is entirely determined by the divergent terms of the partial fraction expansions of $\boldsymbol{\rho}(s)$ and $\boldsymbol{G}(s)$ at $s \to 0$.

Finally, we arrive at analytical expressions for the current from Eq. (4) ($x = \Omega/\Gamma$ and $\gamma = \Gamma'/\Gamma$):

$$I_1=\frac{4x^2(\gamma+1)}{(\gamma+1)^2+12x^2}\Gamma, \tag{8}$$

for the fully interferential case ($\kappa = 1$) and

$$I_0=\frac{(\gamma+4x^2)(\gamma+1)}{(\gamma+1)^2+12x^2}\Gamma, \tag{9}$$

for the fully noninterferential case ($\kappa = 0$). It is found that the current is independent of magnetic flux due to ρ_{00} being constant function of magnetic flux in the strong Coulomb blockade limit. However, the derivation is difficult for the zero-frequency shot noise Eq. (5). We obtain analytical expressions only for several magnetic fluxes: if $\varphi = 0$ and 2π, we have

$$\begin{aligned} S_1(0) \;=\; & 4x^2\Gamma(\gamma+1)[(80\gamma^2+352\gamma+80)x^4 \\ & +(-8\gamma^4+160\gamma^3+336\gamma^2+160\gamma-8)x^2 \\ & +\gamma^6+10\gamma^5+31\gamma^4+44\gamma^3+31\gamma^2+10\gamma+1] \\ & \times(\gamma-1)^{-2}[(\gamma+1)^2+12x^2]^{-3}, \end{aligned} \tag{10}$$

at $\kappa = 1$; while if $\varphi = \pm\pi$ and $\kappa = 1$, we obtain

$$S_1(0) = 4x^2\Gamma(\gamma+1)[80x^4 - (8\gamma^2+16\gamma+8)x^2+\gamma^4 + 4\gamma^3+6\gamma^2+4\gamma+1][(\gamma+1)^2+12x^2]^{-3}. \tag{11}$$

Let's discuss two special situations. When $\Gamma' = 0$ ($\gamma = 0$), the system reduces to series-coupled QDs, and correspondingly, the current and shot noise become

$$I = \frac{4x^2}{1+12x^2}\Gamma, \quad S(0) = \frac{4x^2(80x^4-8x^2+1)}{(1+12x^2)^3}\Gamma, \tag{12}$$

irrespective of κ, which are identical to the previous results.[19] For a completely symmetric interferometer $\Gamma' = \Gamma$ ($\gamma = 1$), the current is

$$I = \begin{cases} \frac{2}{3}\Gamma, & \text{if } \varphi = 0 \text{ and } 2\pi, \\ \frac{8x^2-2\kappa^2+2}{3+12x^2+2\kappa-\kappa^2}\Gamma, & \text{if } \varphi = \pm\pi. \end{cases} \tag{13}$$

While the zero-frequency shot noise becomes

$$S_1(0) = \begin{cases} \frac{1}{3}\Gamma, & \text{if } \varphi = 0 \text{ and } 2\pi, \\ \frac{2x^2(5x^4-2x^2+1)}{(1+3x^2)^3}\Gamma, & \text{if } \varphi = \pm\pi, \end{cases} \tag{14}$$

in the case of full coherence. By contrast, for the full noninterference case, we have a constant shot noise:

$$S_0(0) = \frac{10}{27}\Gamma. \tag{15}$$

For other values of magnetic flux, we have to resort to numerical calculation for shot noise. In Fig. 2, we plot the calculated current I, zero-frequency shot noise $S(0)$, and Fano factor $F = S(0)/I$ with $x = 1/2$ and $\kappa = 1$ as functions of magnetic flux for different coupling γ of the additional pathway in the case of infinite interdot Coulomb repulsion. Our results explicitly show that (1) shot noise and Fano factor are periodic functions of magnetic flux with a period of 2π; (2) shot noise is significantly enhanced due to the destructive quantum interference effect around $\varphi = 0$ and $\pm 2n\pi$ (n is an integer), leading to a giant Fano factor up to 10^2 at $\gamma = 0.8$ for full coherence $\kappa = 1$, while the shot noise is always sub-Poissonian for the system without the additional tunneling path $\gamma = 0$; (3) the instructive quantum interference at $\varphi = \pm n\pi$ (n is an odd integer) suppresses the Fano factor lower than unit, exhibiting sub-Poissonian noise; (4) the inset clearly shows that the giant Fano factor crucially depends on the degree of interference κ. This magnetic-flux-dependent noise property was also reported in the cotunneling transport through a parallel-CQD.[3]

However, the shot noise is always sub-Poissonian in the case of no interdot Coulomb repulsion. In this case, the associated QREs become ($\kappa = 1$)

$$\begin{aligned}
\dot\rho_{11}^{(n)} &= \Gamma\rho_{00}^{(n)} - 2\Gamma'\rho_{11}^{(n)} + \Gamma\rho_{dd}^{(n-1)} + i\Omega(\rho_{12}^{(n)} - \rho_{21}^{(n)}) \\
&\quad -\frac{1}{2}\sqrt{\Gamma\Gamma'}(e^{-i\varphi/2} - e^{i\varphi/2})(\rho_{12}^{(n)} - \rho_{21}^{(n)}), && \text{(16a)} \\
\dot\rho_{22}^{(n)} &= \Gamma'\rho_{00}^{(n)} - 2\Gamma\rho_{22}^{(n)} + \Gamma'\rho_{dd}^{(n-1)} - i\Omega(\rho_{12}^{(n)} - \rho_{21}^{(n)}) \\
&\quad -\frac{1}{2}\sqrt{\Gamma\Gamma'}(e^{-i\varphi/2} - e^{i\varphi/2})(\rho_{12}^{(n)} - \rho_{21}^{(n)}), && \text{(16b)} \\
\dot\rho_{dd}^{(n)} &= \Gamma'\rho_{11}^{(n)} + \Gamma\rho_{22}^{(n)} - (\Gamma + \Gamma')\rho_{dd}^{(n)} \\
&\quad -\sqrt{\Gamma\Gamma'}e^{i\varphi/2}\rho_{12}^{(n)} - \sqrt{\Gamma\Gamma'}e^{-i\varphi/2}\rho_{21}^{(n)}, && \text{(16c)} \\
\dot\rho_{12}^{(n)} &= i\Omega(\rho_{11}^{(n)} - \rho_{22}^{(n)}) - (\Gamma + \Gamma')\rho_{12}^{(n)} \\
&\quad +\sqrt{\Gamma\Gamma'}e^{-i\varphi/2}\rho_{00}^{(n)} - \sqrt{\Gamma\Gamma'}e^{i\varphi/2}\rho_{dd}^{(n-1)} \\
&\quad -\frac{1}{2}\sqrt{\Gamma\Gamma'}(e^{i\varphi/2} - e^{-i\varphi/2})(\rho_{11}^{(n)} + \rho_{22}^{(n)}), && \text{(16d)}
\end{aligned}$$

together with the normalization relation $\rho_{00}+\rho_{11}+\rho_{22}+\rho_{dd} = 1$ [Equation for $\rho_{00}^{(n)}$ is the same as Eq. (1a)], which are quoted directly from our previous derivation using nonequilibrium Green's function.[14] Along the same scheme as above indicated, we obtain the following results: (1) for $\Gamma' = 0$ (the series-coupled QDs),

$$I = \frac{2x^2}{1+4x^2}\Gamma, \quad S(0) = \frac{2x^2(8x^4 - 2x^2 + 1)}{(1+4x^2)^3}\Gamma, \tag{17}$$

which are also identical to the previous results;[19] and (2) for $\Gamma' = \Gamma$ (a completely symmetric interferometer),

$$\begin{array}{lll}
I = \Gamma, & S(0) = \Gamma/2, & \text{if } \varphi = 0 \text{ and } 2\pi, \\
I = \frac{x^2}{1+x^2}\Gamma, & S(0) = \frac{x^2(x^4-x^2+2)}{2(1+x^2)^3}\Gamma, & \text{if } \varphi = \pm\pi.
\end{array} \tag{18}$$

In conclusion, we have analyzed the shot noise properties of resonant tunneling through parallel-coupled QD interferometer at extremely large bias-voltage. Our analytic and numerical results predict that a giant Fano factor can be found in this system due to the combination of quantum interference effect between two tunneling paths and strong Coulomb blockade, and a super-Poissonian–sub-Poissonian transition of the shot noise occurs by tuning the enclosed magnetic flux to change quantum interference pattern.

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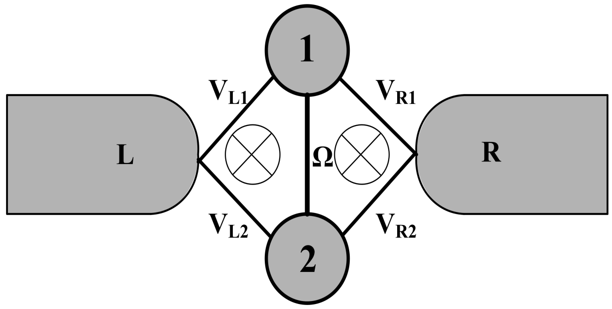
Figure Caption

FIG.1: Schematic diagram for the coherent resonant tunneling through a parallel-coupled double quantum dot Aharonov-Bohm interferometer.

FIG.2: (a) Current I, zero-frequency shot noise $S(0)$ (with unit Γ) and (b) Fano factor $F = S(0)/I$ vs φ for various values of Γ'/Γ with $\Omega/\Gamma = 1/2$ and $\kappa = 1$. Inset: F vs κ.



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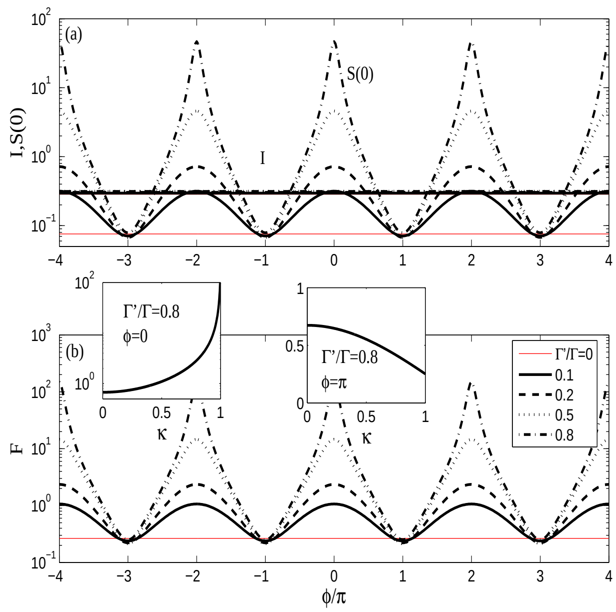

Figure 2: (a) Current I, zero-frequency shot noise $S(0)$ (with unit Γ) and (b) Fano factor $F = S(0)/I$ vs φ for various values of Γ'/Γ with $\Omega/\Gamma = 1/2$ and $\kappa = 1$. Inset: F vs κ.